Two-Region Diffusion Model for Improved Analysis of ADS Experiments

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The 22nd International Conference on Transport Theory
Portland, Oregon, September 12-16, 2011
Motivation

Radiotoxicity and long-term disposal of irradiated nuclear fuel

Around 200,000 m$^2$ of low- and intermediate-level radioactive waste and 10,000 m$^2$ of high-level waste is generated in the world every year.
Accelerator-Driven Systems

- A sub-critical reactor
- A particle accelerator
- Spallation reaction
- A 1000 MeV beam creates 20-30 neutrons per proton
- A 20 mA accelerator can transmute annually the $^{129}$I generated by more than 10 reactors
Accelerator-Driven Systems

On-line monitoring of the reactivity level in an accelerator-driven system is of major importance for safe operation!
Yalina-Booster Experiment
Yalina-Booster Core

- **Fast zone**: 36% and 90% enriched UO$_2$ in lead

- **Valve zone**: natural uranium and B$_4$C pins

- **Thermal zone**: 10% enriched UO$_2$ in polyethylene

- **Graphite reflector**

- **B$_4$C control rods**
Results of the Experiments

Calle Berglöf, 2nd EROTRANS Progress Meeting, 2009, France
Comparison of Analytical Solutions with Experiments

B. Merk, V. Glivici-Cotruţă, F.P. Weiß, XXI International Conference on Transport Theory, 2009, Italy
Comparison with Experiments

Results for the fast zone:

- ✓ good reproduction of the detector response
- ✓ good agreement between diffusion and $P_1$ transport
- ✓ $k_{eff} \approx 0.6$ for the fast zone only
Green’s Function for the Diffusion Equation in a Composite Slab

\[
\frac{1}{v_1} \frac{\partial \Phi_1(x, t)}{\partial t} - D_1 \frac{\partial^2 \Phi_1(x, t)}{\partial x^2} + \sum_{a_1} \Phi_1(x, t) = \delta(x + \xi) \delta(t - \tau), \quad -a < x < 0, \quad t > 0
\]

\[
\frac{1}{v_2} \frac{\partial \Phi_2(x, t)}{\partial t} - D_2 \frac{\partial^2 \Phi_2(x, t)}{\partial x^2} + \sum_{a_2} \Phi_2(x, t) = 0, \quad 0 < x < b, \quad t > 0
\]

\(a, b, \xi, \tau > 0\)

Initial conditions:

\[
\Phi_1(x, 0) = 0, \quad -a < x < 0
\]

\[
\Phi_2(x, 0) = 0, \quad 0 < x < b
\]
Green’s Function for the Diffusion Equation in a Composite Slab

Boundary conditions:
\[
\frac{\partial \Phi_1 (x, t)}{\partial x} \bigg|_{x=-a} = 0, \quad t > 0
\]
\[
\frac{\partial \Phi_2 (x, t)}{\partial x} \bigg|_{x=b} = 0, \quad t > 0
\]

Conditions at the interface:
\[
\Phi_1 (0, t) = \Phi_2 (0, t), \quad t > 0
\]
\[
D_1 \cdot \frac{\partial \Phi_1 (x, t)}{\partial x} \bigg|_{x=0} = D_2 \cdot \frac{\partial \Phi_2 (x, t)}{\partial x} \bigg|_{x=0}, \quad t > 0
\]
Geometry
Green’s Function for the Diffusion Equation in a Composite Slab

Notation: \[
\frac{1}{\nu_i D_i} = \frac{1}{k_i}, \quad \sum_{a_i} = c_i, \quad i = 1, 2
\]

Substitution: \[
\Phi_i = u_i e^{-k_i c_i t}, \quad i = 1, 2
\]

\[
\frac{\partial^2 u_1(x, t)}{\partial x^2} - \frac{1}{k_1} \frac{\partial u_1(x, t)}{\partial t} = -\frac{e^{k_1 c_1 t}}{D_1} \delta(x + \xi) \delta(t - \tau),
\]

\[-a < x < 0, \quad t > 0
\]

\[
\frac{\partial^2 u_2(x, t)}{\partial x^2} - \frac{1}{k_2} \frac{\partial u_2(x, t)}{\partial t} = 0,
\]

\[0 < x < b, \quad t > 0\]
Green’s Function for the Diffusion Equation in a Composite Slab

Initial conditions:
\[ u_1(x,0) = 0, \quad -a < x < 0 \]
\[ u_2(x,0) = 0, \quad 0 < x < b \]

Boundary conditions:
\[ \frac{\partial u_1(x,t)}{\partial x} e^{-k_1c_1 t} \bigg|_{x=-a} = 0, \quad t > 0 \]
\[ \frac{\partial u_2(x,t)}{\partial x} e^{-k_2c_2 t} \bigg|_{x=b} = 0, \quad t > 0 \]

Conditions at the interface:
\[ u_1(0,t)e^{-k_1c_1 t} = u_2(0,t)e^{-k_2c_2 t}, \quad t > 0 \]
\[ D_1 \cdot \frac{\partial u_1(x,t)}{\partial x} e^{-k_1c_1 t} \bigg|_{x=0} = D_2 \cdot \frac{\partial u_2(x,t)}{\partial x} e^{-k_2c_2 t} \bigg|_{x=0}, \quad t > 0 \]
Green’s Function for the Diffusion Equation in a Composite Slab

Laplace transform:

\[
\frac{\partial^2 G_1}{\partial x^2} - \frac{s}{k_1} G_1 = - \frac{e^{-(s-k_1c_1)\tau}}{D_1} \delta(x + \xi), \quad -a < x < 0
\]

\[
\frac{\partial^2 G_2(x,t)}{\partial x^2} - \frac{s}{k_2} G_2(x,t) = 0, \quad 0 < x < b
\]

Boundary conditions:

\[
\frac{1}{s + k_1c_1} \frac{\partial G_1}{\partial x} \Big|_{x=-a} = 0
\]

\[
\frac{1}{s + k_2c_2} \frac{\partial G_2}{\partial x} \Big|_{x=b} = 0
\]
Green’s Function for the Diffusion Equation in a Composite Slab

Conditions at the interface:

\[
\frac{1}{s + k_1 c_1} G_1(0, s) = \frac{1}{s + k_2 c_2} G_2(0, s)
\]

\[
D_1 \left[ \frac{1}{s + k_1 c_1} \cdot \frac{\partial G_1(x, s)}{\partial x} \right]_{x=0} = D_2 \left[ \frac{1}{s + k_2 c_2} \cdot \frac{\partial G_2(x, s)}{\partial x} \right]_{x=0}
\]

Notation:

\[
\beta^2 = -\frac{s}{k_1}, \quad \lambda^2 = \frac{k_1}{k_2}
\]
Green’s Function for the Diffusion Equation in a Composite Slab

\[
G_1(x, s) = \frac{1}{D_1\beta} e^{-(s-k_1c_1)\tau} (-H(x + \xi) \sin \beta(x + \xi) + \frac{\lambda D_2 \sin \lambda \beta b \sin \beta \xi - D_1 \cos \lambda \beta b \cos \beta \xi}{\lambda D_2 \sin \lambda \beta b \cos \beta a + D_1 \cos \lambda \beta b \sin \beta a} \cos(x + a))
\]

\[
G_2(x, s) = -\frac{s + k_2c_2}{s + k_1c_1} \cdot \frac{e^{-(s-k_1c_1)\tau} \cos \beta(\xi - a) \cos \lambda \beta(x - b)}{\beta(\lambda D_2 \sin \lambda \beta b \cos \beta a + D_1 \cos \lambda \beta b \sin \beta a)}
\]
Green’s Function for the Diffusion Equation in a Composite Slab

\[ u_1(x,t) = \frac{1}{2\pi i} \int_{\gamma-i\infty}^{\gamma+i\infty} \frac{1}{D_1 \beta} e^{-\left(s-k_1 c_1\right)\tau} \left(-H(x + \xi)\sin \beta(x + \xi) + \frac{\lambda D_2 \sin \lambda \beta b \sin \beta \xi - D_1 \cos \lambda \beta b \cos \beta \xi}{\lambda D_2 \sin \lambda \beta b \cos \beta a + D_1 \cos \lambda \beta b \sin \beta a} \cos(x + a)\right) ds \]

Poles are:

\[ s = 0 \]
\[ s = -k_1 c_1 \]
\[ s_n = -k_1 \beta_n^2 \], where the residues are given by \( \frac{p(s_n)}{q'(s_n)} \)
Green’s Function for the Diffusion Equation in a Composite Slab

\[ u_2(x, t) = \frac{-2k_2c_2e^{k_1c_1\tau}}{D_1c_1(aD_1 + \lambda^2bD_2)} + \]

\[ + \frac{(-k_1c_1 + k_2c_2)\cos\sqrt{c_1}(\xi - a)\cos\lambda\sqrt{c_1}(x - b)}{k_1\sqrt{c_1}(D_1\sin\sqrt{c_1}a\cos\lambda\sqrt{c_1}b + \lambda D_2\sin\sqrt{c_1}b\cos\sqrt{c_1}a)} \]

\[-\frac{s_n + k_2c_2}{(s_n + k_1c_1)} \times \]

\[-\frac{e^{-(s_n - k_1c_1)\tau}\cos\beta_n(\xi - a)\cos\lambda\beta_n(x - b)}{\cos\lambda\beta_n b\cos\beta_n a(aD_1 + \lambda^2bD_2) + (-\lambda bD_1 - \lambda aD_2)\sin\lambda\beta_n b\sin\beta_n a} \]

\]
Summary and Conclusions

✓ Full analytical solution for the $P_1$ and diffusion equations with external source is developed with Green’s function method

✓ Comparison with experimental results from YALINA-Booster

✓ Evaluation of difference between time dependent $P_1$ transport and diffusion

✓ Solution of a problem for the two regions diffusion equation with appropriate boundary conditions for YALINA experiments
Further Steps

✓ Development of something comparable with widely used integral reactor physical parameters like “corrected reactivity”

✓ Development for an analytical solution for the shut down of the source

✓ Evaluation of the mathematical model for the Guinevere experiment
Thank you for your attention!