Analytical discrete ordinate method for radiative transfer in vegetation canopies

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Outline

- Introduction
- Radiative transfer model for canopies
  - Fundamental equations
  - Peculiarities of RT in vegetation
- Analytical Discrete Ordinate method for photon transport in canopies
- Results
- Conclusions & perspectives
Introduction

- Photon transport in dense vegetation can be described by means of Boltzmann equation if coherent scattering is disregarded.

- Main assumptions:
  - No photon/photon interaction $\Rightarrow$ linearity
  - Remote sensing observation $\Rightarrow$ 1D problem
  - Relative direction b/w sun and observ. angle $\Rightarrow$ two-angle formulation
  - No frequency-shift $\Rightarrow$ hyperspectral solution through uncoupled monoenergetic problems
  - Polarization effects (here disregarded)
Applications

- The study of the reflected light from a vegetated region can be interesting for:
  1. Remote sensing
     - E.g. detection of objects under canopies for defense and security reasons
  2. Mapping status of vegetation
     - E.g. agricultural purposes and for mapping plant physiology

=> Need of efficient and accurate simulation tools to interpret experimental measurements
Problem setting

Features:
- Dense vegetation
- Leaf as a point scatterer
- Scattering is not rotationally-invariant
- Absorption is angular dependent
Scattering features

1. Isotropic medium with anisotropic scattering

Rotationally-invariant scattering:
\[ \sigma_s (\Omega' \rightarrow \Omega) = f(\Omega \cdot \Omega') \]

2. Anisotropic medium

Scattering specifically depends on inward and outward directions:
\[ \sigma_s (\Omega' \rightarrow \Omega) = f(\Omega', \Omega) \]
Total cross section definition

\[ \sigma(z, \Omega) = G(\Omega)u_L(z) \]

**leaf density**

**geometric factor**

optical thickness: \( \tau(z) = \int_0^z dz' u_L(z') \)

average over leaf normal distribution

\[ G(\Omega) = \frac{1}{2\pi} \int_0^{2\pi} d\phi_L \int_0^1 d\mu_L g(\mu_L, \phi_L) |\Omega_L \cdot \Omega| \]

leaf angle distribution

relative angle between leaf and propagation direction

less area intercepted

stronger attenuation
Fundamental equations

- Transport equation for anisotropic media:

\[
- \mu \frac{\partial I(\tau, \Omega)}{\partial \tau} + G(\Omega) I(\tau, \Omega) = \int d\Omega' \Gamma(\Omega' \rightarrow \Omega) I(\tau, \Omega')
\]

where:

\[
\Gamma(\Omega' \rightarrow \Omega) = \frac{1}{2\pi} \int_{0}^{+1} d\mu_L g_L(\Omega_L) |\Omega_L \cdot \Omega'| f(\Omega' \rightarrow \Omega; \Omega_L)
\]

- leaf angle distribution
- relative angle between leaf and propagation direction
- for a given leaf with \( \Omega_L \), probability of deviation of particle with direction \( \Omega' \) in direction \( \Omega \)
- average over leaf normal distribution
Boundary conditions

- rigorous b.c. would include a transport model in air and in soil
- approximate b.c. can be expressed as a localized+diffuse conditions at the top of canopy (TOC) and Lambertian reflection on the bottom of canopy (BOC)

\[ I(0, \Omega) = S_0 \delta(\Omega - \Omega_0) + S_d(\Omega) \]
\[ I(\Delta, \Omega) = \frac{I_S}{\pi} \int_0^{2\pi} \int_0^1 d\phi' d\mu' \mu' |I(\Delta, \Omega')| \]
\[ \Omega'(\mu', \phi') \]
Canopy structure

- Directionality of leaves
  - The normal directions associated to each leaf are distributed according to the leaf angle distribution (LAD)
  - LAD varies according to the vegetation species
    - planophiles tend to have horizontal leaves (e.g. oaks)
    - erectophile, vertical development (e.g. grasses)
  - LAD can also vary along depth of canopy

![Graph of leaf angle distribution](image)
Leaf model

- Leaf as a bi-Lambertian surface

- Scattering properties of leaves are characterized by the reflectance (i.e. $r_L$) and transmittance (i.e. $t_L$) as suggested by Ross (1981).

- Two-level transport model (Ganapol et al., 1992)
  1. leaf model: from leaf features, find $r_L$ and $t_L$
  2. canopy model: given $r_L$ and $t_L$, find canopy reflectance and transmittance

\[ f(\Omega' \rightarrow \Omega; \Omega_L) = \begin{cases} 
  r_L|\Omega \cdot \Omega_L|, & (\Omega \cdot \Omega_L)(\Omega' \cdot \Omega_L) < 0 \\
  t_L|\Omega \cdot \Omega_L|, & (\Omega \cdot \Omega_L)(\Omega' \cdot \Omega_L) > 0 
\end{cases} \]
Leaf model

- Comparison of phase functions

- Symmetries of the scattering kernel analyzed by Picca and Furfaro (2009) in ICTT-21
Solution techniques

- Several techniques were tested in the past
  - Integral method
  - 4-flux method
  - Case’s method
  - $S_N$ method
  - Converged $S_N$ method
  - ...

- Problems:
  - Hypothesis and limitations
  - Accuracy
  - Computational time
Analytical Discrete Ordinate method

- Basic principle of ADO
  - Developed by Chandrasekhar and recently reformulated by Siewert et al.
  - Semi-analytical method (in space)
  - Angle variable is discretized ($S_N$)
  - No iteration needed for scattering term
  - Suitable for isotropic media (possibly with anisotropic scattering)
  - Two-angle solution is possible considering the Fourier expansion of the solution (one mode per Fourier component, uncoupled eqs)
Canopy analytical discrete ordinate

- Basic equations for canopies re-arranged
  - Equations for intensities in the + directions separated from that in the − directions
  - Solution in the form: \( \bar{I}_{\pm}(\tau) = \Phi_{\pm}(\nu)e^{\mp \tau/\nu} \)

- For generic cross sections and scattering function, the ADO techniques cannot be apply
  - Eigenvalue problem comes out from a combination of the two-eqs...
Canopy analytical discrete ordinate

Symmetries in p.f. and xsec definitions for canopies

\[
\frac{1}{\nu} \hat{M} \left[ \Phi_+ - \Phi_- \right] = \hat{G} \left[ \Phi_+ + \Phi_- \right] - \left( \hat{\Pi}_+ + \hat{\Pi}_- \right) \left[ \Phi_+ + \Phi_- \right] \\
\frac{1}{\nu} \hat{M} \left[ \Phi_+ + \Phi_- \right] = \hat{G} \left[ \Phi_+ - \Phi_- \right] - \left( \hat{\Pi}_+ - \hat{\Pi}_- \right) \left[ \Phi_+ - \Phi_- \right]
\]

Sum and subtractions of original eqs.

\[
\bar{X} = \hat{M} \left[ \Phi_+ + \Phi_- \right]
\]

Eigenvalue problem

\[
\frac{1}{\nu^2} \bar{X} = \hat{F} \hat{E} \bar{X}
\]

\[
\hat{E} = \left( \hat{G} - \hat{\Pi}_+ - \hat{\Pi}_- \right) \hat{M}^{-1}
\]

\[
\hat{F} = \left( \hat{G} - \hat{\Pi}_+ + \hat{\Pi}_- \right) \hat{M}^{-1}
\]
Canopy analytical discrete ordinate

Complete solution (homogeneous+particular)

\[
\begin{align*}
\vec{I}_+ (\tau) &= \sum_{n=1}^{N} \left[ A_n \Phi_{+,n} e^{-\tau/n} + B_n \Phi_{-,n} e^{-(\tau_0-\tau)/n} \right] + \vec{I}_+^p (\tau) \\
\vec{I}_- (\tau) &= \sum_{n=1}^{N} \left[ A_n \Phi_{-,n} e^{-\tau/n} + B_n \Phi_{+,n} e^{-(\tau_0-\tau)/n} \right] + \vec{I}_-^p (\tau)
\end{align*}
\]

particular solution
(Barichello et al., 2000)

Imposing b.c. to determine coeffs \( A_n \) and \( B_n \)

\[
\begin{align*}
\sum_{n=1}^{N} \left[ A_n \Phi_{+,n} + B_n \Phi_{-,n} e^{-\tau_0/n} \right] &= -\vec{I}_+^p (0) \\
\sum_{n=1}^{N} \left[ A_n \Phi_{-,n} e^{-\tau_0/n} + B_n \Phi_{+,n} \right] &= -\vec{I}_-^p (\tau_0) + L \hat{I}_N
\end{align*}
\]

\[
L = 2r_s \hat{W} \hat{M} \vec{I}_+ (\tau_0) = 2r_s \hat{W} \hat{M} \left\{ \sum_{n=1}^{N} \left[ A_n \Phi_{+,n} e^{-\tau_0/n} + B_n \Phi_{-,n} \right] + \vec{I}_+^p (\tau_0) \right\}
\]
Result summary

- Benchmark results
  - Comparison with SN and FN
- Hyperspectral results
  - LOPEX database
- Bi-directional reflectance and transmittance
Benchmark results (I)

Reflectance & transmittance intensities

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**Input:**

- $\theta_0 = 0.0^\circ$
- $\theta^* = 25.31^\circ$
- $r_s = 0.2$
- $\Delta = 1.0$
- $\tau_L = 0.5$
- $\rho_L = 0.4$

**FN and SN from Ganapol and Myneni (1992)**

At least 3-digit agreement
### Benchmark results (II)

#### Reflectance & transmittance

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</table>

#### Input:
- \( \Delta = 1.0 \)
- \( r_s = 0.2 \)
- A. \( \rho_L = 0.07 \) \( \tau_L = 0.03 \)
- B. \( \rho_L = 0.45 \) \( \tau_L = 0.45 \)

At least 4-digit agreement

*FN from Ganapol and Myneni (1992)*
Hyperspectral results

Input:
LAI = 10, \( r_s = 0.0 \)
oaks (planophile)
\( \rho_L, \tau_L \) from LOPEX database
(Hosgood et al., 1995)
Results for single-oriented leaves (I)

Angular distribution of reflectance

Input:
LAI = 1, \( r_s = 0.0 \)
1. \( \theta^* = 10^\circ \)
3. \( \theta^* = 60^\circ \)
4. \( \theta^* = 85^\circ \)

mean angle of the single-oriented leaves

TOC
BOC
Results for single-oriented leaves (II)

Transmittance angular distribution

Input:
LAI = 1, \( r_s = 0.0 \)
1. \( \theta^* = 10^\circ \)
3. \( \theta^* = 60^\circ \)
4. \( \theta^* = 85^\circ \)
Conclusions and perspectives

- We presented a review of peculiar features of RT in vegetation compared to classical transport in isotropic media.
- The ADO method can be extended to deal with anisotropic media only if certain symmetries hold.
- Results are compared with literature and are in excellent agreement.
- Future work is foreseen to extend the technique to the two-angle case.
Backup
Symmetry properties

- RT in canopies:

\[ \Gamma(\Omega' \rightarrow \Omega) = \Gamma(\Omega \rightarrow \Omega') = \Gamma(-\Omega' \rightarrow -\Omega) \]
Canopy structure properties

- LAD can normally be expressed assuming polar and azimuthal angles are independent
  \[ g_L(\Omega_L) = h_L(\phi_L)k(\mu_L) \]

- A random leaf distribution in azimuth is experimentally observed => \( h_L(\phi_L) = 1 \)

\[
G(\Omega) = \int_0^{+1} d\mu_L k_L(\mu_L)\psi(\mu, \mu_L)
\]

\[
\psi(\mu, \mu_L) = \frac{1}{2\pi} \int_0^{2\pi} d\phi_L |\Omega \cdot \Omega_L|
\]

analytical derivation of total cross section (Shultis and Myneni)
Phase function definition

Area transfer function definition

\[ \Gamma(\Omega' \rightarrow \Omega) = r_L \Gamma^- (\Omega' \rightarrow \Omega) + t_L \Gamma^+ (\Omega' \rightarrow \Omega) \]

where:

\[ \Gamma^\pm (\Omega' \rightarrow \Omega) = \pm \frac{1}{2\pi} \int_{0}^{+1} d\phi_L \int_{\pm 0}^{2\pi} d\mu_L g_L (\Omega_L) (\Omega_L \cdot \Omega')(\Omega_L \cdot \Omega) \]

being the sign “+” relative to the values for which integrand is positive and “-” for negative.
Leaf model

- Assumptions:
  - non-dimensional planar scattering center
  - spatially non correlated with one another (otherwise MC, ray-tracing, stochastic approaches)

- Scattering model choice is not trivial...
  - Isotropic scattering
  - Anisotropic rotationally-invariant phase function (such as Henyey-Greenstein)

=> *Experimental measurements show that using a rotationally-invariant phase function introduces non-negligible errors*